ON FIBERINGS WITH SINGULARITIES

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1. INTRODUCTION

The original treatment of fiberings goes back to H. Seifert [5] who permitted a certain type of singularity: namely, the case where neighboring fibers wind themselves around a particular fiber. About a dozen years later, in 1945, Montgomery and Samelson [4] studied a different type of singularity which may be roughly described by saying that certain fibers are pinched to points.

In the present paper, we shall first introduce a general definition of fiberings with singularities which includes all of the known fiberings as special cases. Then, by identifying the singular fibers to points, we shall prove that every fibering p with singularities can be decomposed into the composition $p_1 \circ p_2$ of two fiberings p_1 and p_2, where every singular fiber of p_1 is a singleton and every regular fiber of p_2 is a singleton. Furthermore, p_2 is the natural projection of some topological identification. Therefore, after that point, we shall be concerned only with fiberings whose singular fibers are singletons.

As an important and useful fibering with only one singular fiber, we shall introduce and study the extended tangent space $E(X, x_0)$ of a given space X at a given point x_0 and the natural projection $p: E(X, x_0) \to X$. The invariants of the regular fibers of this fibering are closely related with the invariants of the residual space $X \setminus x_0$ and the local invariants of X at x_0.

Our main interest is to investigate the local property of any given fibering $f: X \to Y$ at an isolated singular fiber x_0. For this purpose, we may assume without loss of generality that x_0 is the only singular fiber of f. If $y_0 = f(x_0)$, then the map $f:(X, x_0) \to (Y, y_0)$ induces homomorphisms on the local homology groups and the local homotopy groups defined in [2]. To study these induced homomorphisms, we shall investigate the derived continuous map $\hat{f}: E(X, x_0) \to E(Y, y_0)$. Under a mild condition on f, this map $\hat{f}$ is a fibering with only one singular fiber. We shall say that f is normal at x_0 if and only if $\hat{f}$ is a fibering with only one singular fiber. We shall prove that the *fibering axiom* of the local homotopy groups holds for every fibering with a single normal singular fiber [2, Section 16].

The invariants of the regular fibers $\hat{F}$ of the derived fibering $\hat{f}$ are closely related to the local invariants of the spaces X and Y at the points x_0 and y_0, respectively. If Y is locally euclidean, then we have a local version of Wang's exact sequence [7] which will help determine the homology groups of $\hat{F}$.

Finally, by a cone construction, it will be shown that the usual global theory of fiberings without singularities may be deduced from a special case of the local theory of fiberings at an isolated singularity.

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