QUANTIFIER ELIMINATION IN A PROBLEM OF LOGICAL DESIGN

Calvin C. Elgot and Jesse B. Wright

Abstract. A fundamental problem in the theory of logical design is that of expressing behavior realizable by computer circuits. For time-independent circuits, formulas of the propositional calculus have been helpful in expressing behavior. For time-dependent circuits, attempts have been made to use, for this purpose, formulas in which time variables may be quantified.

By the method of quantifier elimination, we find the expressive power of a certain class of formulas which has been used in attempts to describe circuit behavior. We then prove the inadequacy of these formulas for expressing a kind of computer behavior.

1. INTRODUCTION

In the theory of logical design it is important to distinguish the behavior of a computer circuit from its structure. The behavior of a computer circuit is the relationship between every sequence of input signals and the resulting output signals, while the structure is the particular pattern of connections of the components which effect this relationship. In order to deal with some of the fundamental problems in logical design, such as synthesis and analysis, it is necessary to employ some systematic method of symbolizing behavior. Propositional calculus has long been used, often indiscriminately, for describing the structure or the behavior of circuits of the combinational or time-independent type. In the case of sequential circuits, the dependence of the output signals on the complete past history of input signals suggests the use of formulas with quantified time variables for expressing behavior. The discussion in this paper concerns certain sets of formulas in the first-order monadic predicate calculus.

In order to state and prove rigorously results about the use of formulas in expressing circuit behavior, it is first necessary to define precisely what it means for a formula to express the behavior of a circuit. Circuits are labeled by assigning to each input a distinct monadic predicate variable. An output p of a circuit *realizes* a formula $F(I_1, \cdots, I_n, t)$ if, for every assignment of propositional functions $\phi_1, \cdots, \phi_k$ to the predicate variables $I_1, \cdots, I_n$ and of a specific time t_0 to the variable t:

if (a) for all i and x, $\phi_i(x)$ if and only if input ϕ_i is active at time x,

then (b) $F(\phi_1, \cdots, \phi_n, t_0)$ if and only if p is active at time t_0.

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