# A NOTE ON A THEOREM OF H. KNOTHE

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In this journal, H. Knothe proved two "inverse Archimedean theorems" for convex bodies in Euclidean 3-space [1]. The present note extends the first of these two theorems and thereby furnishes a somewhat simpler proof of the first theorem itself.

Let $C(u)$ be the cylinder whose generators have the direction $u$ and which circumscribes the convex body $K$. Further, let $B(u)$ be the breadth of $K$ in the direction $u$, and $L(u)$ the perimeter of the projection of $K$ in the direction $u$. Since $B(u)$ is continuous over the sphere of directions $u$, it has a maximum, say at $u_0$, and a minimum, say at $u_1$. Finally, denote by $S(u)$ the lateral area of that part of $C(u)$ which lies between the support planes of $K$ in the directions $u$ and $-u$.

THEOREM 1. *If* $S(u)$ *is constant,* $K$ *is of constant breadth.*

We first note that, for any $u$,

$$\pi B(u_0) \geq L(u) \geq \pi B(u_1)$$

because

$$L(u) = \pi \overline{B}(u)\,, \tag{1}$$

where $\overline{B}(u)$ is the arithmetic mean of breadths orthogonal to $u$. Therefore

$$S(u_0) = B(u_0)\, L(u_0) \geq \pi B(u_0)\, B(u_1) \tag{2}$$

and

$$S(u_1) = B(u_1)\, L(u_1) \leq \pi B(u_1)\, B(u_0)\,. \tag{3}$$

Since

$$S(u_0) = S(u_1) = S(u)$$

for any direction $u$, we have, from the second parts of (2) and (3),

$$S(u) = \pi B(u_0)\, B(u_1)\,.$$

Then, by the first part of (2),

$$L(u_0) = \pi B(u_1)\,.$$

Because $B(u_1)$ is a minimum of $B(u)$, it follows from (1) that

$$B(u) = B(u_1) \tag{4}$$

whenever $u$ and $u_0$ are orthogonal.

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