

# ON NORMAL AND EPr MATRICES

Martin Pearl

## 1. INTRODUCTION

H. Schwerdtfeger [2] has called a matrix  $A$  of order  $n$  and rank  $r$  with elements from the complex field  $C$  an EPr matrix if it satisfies the condition

$$(1) \quad \sum_{i=1}^n \alpha_i A_i = 0 \quad \text{if and only if} \quad \sum_{i=1}^n \bar{\alpha}_i A^i = 0 \quad (\alpha_i \in C),$$

where  $A_i$  is the  $i$ th row of  $A$  and  $A^i$  is the  $i$ th column of  $A$ . Another formulation of this definition is that

$$(1') \quad A\xi = 0 \quad \text{if and only if} \quad A^*\xi = 0,$$

where  $\xi$  is contained in the complex  $n$ -dimensional Euclidean space  $C_n$ . The class of EPr matrices ( $r = 0, 1, \dots, n$ ) contains the normal matrices and the nonsingular matrices as subclasses.

In Section 2, other characterizations of complex EPr matrices are given, and in Section 3, EPr matrices are used to develop characterizations for normal matrices with elements from an arbitrary field.

## 2. COMPLEX EPr MATRICES

**THEOREM 0** (Schwerdtfeger [2, p. 131]). *A necessary and sufficient condition that  $A$  be an EPr matrix is that there exist a nonsingular matrix  $Q$  such that  $QAQ^*$  is the direct sum of a nonsingular matrix  $D$  of order  $r$  and a zero matrix.*

The following theorem gives other necessary and sufficient conditions that  $A$  be an EPr matrix.

**THEOREM 1.** *The following statements are equivalent:*

- (i)  $A$  is an EPr matrix;
- (ii)  $A$  is unitarily similar to the direct sum of a nonsingular matrix  $D$  of order  $r$  and a zero matrix;
- (iii)  $A$  is congruent to the direct sum of a nonsingular matrix  $D$  of order  $r$  and a zero matrix;
- (iv)  $A$  is the matrix of a linear transformation  $T$  acting on  $C_n$  and having the property that  $C_n$  can be expressed as the direct sum of two mutually orthogonal  $T$ -spaces  $V_1$  and  $V_2$  such that  $T(V_1) = V_1$ ,  $T(V_2) = 0$  and  $V_1$  has dimension  $r$ ;

---

Received August 16, 1957.

This work was completed under a National Research Council-National Bureau of Standards Postdoctoral Research Associateship.