ON THE SHEETED STRUCTURE OF COMPACT LOCALLY AFFINE SPACES

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INTRODUCTION

Let M^n be an n-dimensional, compact, locally affine space; that is, let M^n carry a complete affine connection with curvature and torsion tensors equal to zero. It is well known (see [1]), that any locally affine space can be realized in the following manner. Let Γ be the fundamental group of M^n. Then the affine connection on M^n determines an imbedding of Γ in the group $A(n)$ of affine transformations of the n-dimensional affine space A^n. Further, the orbit space A^n/Γ is homeomorphic to M^n. Let T denote the subgroup of Γ consisting of all pure translations. Then T is a free abelian group on a finite number of generators. Let $h(\Gamma) = \Gamma/T$. Then $h(\Gamma)$ is called the holonomy group of Γ. The purpose of this paper is to prove the following three theorems:

THEOREM 1. *Let* T *be a free abelian group on* s *generators* ($s \geq 1$). *Assume also that* $h(\Gamma)$ *contains no elements of finite order. Then* M^n *is a fiber bundle over a compact locally affine space* X *with the* s*-dimensional torus as fiber. Further, the fundamental group of* X *is isomorphic to* $h(\Gamma)$.

THEOREM 2. *Let* T *be a free abelian group on* s *generators* ($s \geq 1$). *Then there exists a mapping* $p: M \rightarrow X$, *where* X *is a compact space (not necessarily a manifold) with the following properties:*

I. *For all* $x \in X$, $p^{-1}(x)$ *is a compact,* s*-dimensional manifold which can be given a Riemann metric with zero curvature and torsion.*

II. *The mapping* p *satisfies the hypothesis required for applying the Fáry spectral sequence* (see [2]).

In [3], Zassenhaus defined the radical R of a discrete matrix group Γ as the maximal solvable normal subgroup, and he proved that R is unique.

THEOREM 3. *Let* Γ *be the fundamental group of a compact locally affine space* M, *and assume that* Γ *has a nontrivial radical. Then there exists a mapping* $p: M \rightarrow X$, *where* X *is a compact space (not necessarily a manifold) and the preimage of each point of* X *under* p *is a compact manifold with a torus as covering space.*

The paper concludes with an example of a locally affine manifold which satisfies the hypothesis of Theorem 2, but not the hypothesis of Theorem 1.

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