SIMPLE WAVES IN THE STEADY ROTATIONAL PLANE SUPERSONIC FLOW OF A POLYTROPIC GAS OF CONSTANT ENTROPY

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1. INTRODUCTION

By use of the intrinsic form of the characteristic relations [1], we study the problem of determining all plane, steady rotational flows possessing ∞^1 straight-line bicharacteristics. We call these flows *simple waves*, and we investigate their properties.

For all γ (where γ is the ratio of the specific heats of the gas), two types of simple waves are shown to exist: (1) flows in which the simple waves are radial lines; (2) flows in which the simple waves are the ∞^1 lines tangent to a circle. These flows are vortex flows. Expressions for the speed of sound, the magnitude of the velocity, and the vorticity are determined. In the case of $\gamma = 3$, the above two types of simple waves are found to be the *only* simple waves. Finally, all flows at Mach number one are shown to be centered simple waves.

2. THE BASIC RELATIONS

The basic relations were derived in a previous paper [1]. Here, we shall summarize them briefly. References to equations in [1] will be starred.

Let x^j $(j = 1, 2, 3)$ denote a Cartesian orthogonal coordinate system in Euclidean three-space, and let us denote partial derivatives by the symbolism

$$\partial_j = \frac{\partial}{\partial x^j} .$$

In a Cartesian orthogonal system, covariant and contravariant quantities are equivalent. However, in order to use the Einstein summation convention of summing on repeated upper and lower indices, we shall use both of these quantities. Further, we introduce the following scalars, vectors and tensors:

q, the magnitude of the velocity vector;

c, the local speed of sound;

$b = (q^2 - c^2)^{1/2}$;

t^j, the unit tangent vector along a bicharacteristic curve;

n^j, the unit normal vector of a characteristic surface;

p^j, a unit vector orthogonal to t^j, n^j and such that the ordered orthogonal triple p^j, t^j, n^j forms a right-handed set;

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