A UNIQUENESS THEOREM ON TWO-DIMENSIONAL RIEMANNIAN MANIFOLDS WITH BOUNDARY

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1. INTRODUCTION

The purpose of this paper is to establish the following

THEOREM. *Let* M_2 *and* M_2^* *be two oriented two-dimensional Riemannian manifolds of class* C^2 *imbedded in a Euclidean space* E_{N+2} *of dimension* $N+2$ $(N > 0)$, *with boundaries* C *and* C*, *respectively, and with positive Gaussian curvatures in every normal direction. Suppose that there exists an orientation-preserving differentiable homeomorphism* H *of the manifold* M_2 *onto the manifold* M_2^* *such that at corresponding points the manifolds* M_2 *and* M_2^* *have parallel tangent planes and equal sums of the principal radii of curvature associated with every common normal direction. If the homeomorphism* H *restricted to the boundary* C *is a translation (strictly speaking: is induced by a translation in the space* E_{N+2}*) carrying the boundary* C *onto the boundary* C*, *then the homeomorphism* H *is a translation carrying the whole manifold* M_2 *onto the whole manifold* M_2^*.

For the case where $N = 1$ and the boundaries C and C* of the two manifolds are empty, this theorem was proved by Christoffel [3]; for the general case where $N = 1$, it is due to the author [4]. The method used in this paper is an extension of that used by Chern [2] in proving the uniqueness theorem for Minkowski's problem for closed convex surfaces imbedded in a three-dimensional Euclidean space. (We recall that, by a result of Nash [6] on C^3 isometric imbeddings, every two-dimensional Riemannian manifold with a C^3 positive metric can be imbedded in some Euclidean space.)

2. PRELIMINARIES

Let M_2 be an orientable two-dimensional Riemannian manifold of class C^3 imbedded in a Euclidean space E_{N+2} of dimension $N+2$ $(N > 0)$. To avoid confusion, we shall use the following ranges of indices throughout this paper:

$$\alpha, \beta = 1, 2; \quad 3 \le r \le N+2; \quad 1 \le i, j, k \le N+2. \tag{2.1}$$

Associated with a point P in the space E_{N+2} we introduce a right-handed rectangular frame $Pe_1 \cdots e_{N+2}$ such that $e_1, \cdots, e_{N+2}$ form an ordered set of mutually perpendicular unit vectors with the determinant $(e_1, \cdots, e_{N+2})$ equal to +1. Let X denote the position vector of the point P with respect to a fixed point O in the space E_{N+2}; then we can write

$$dX = \sum_i \omega_i e_i, \qquad de_i = \sum_j \omega_{ij} e_j, \tag{2.2}$$

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