THE FOURIER COEFFICIENTS OF AUTOMORPHIC FORMS BELONGING TO A CLASS OF HOROCYCLIC GROUPS

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1. R. A. Rankin has recently coined the term "horocyclic group" as an English equivalent of the French "Fuchsian group of the first kind" and the German "*Grenzkreisgruppe*" ([8]). He calls such a group "real" if all substitutions of the group preserve the real axis, and "zonal" if the group contains translations. In this paper, we shall refer to real zonal horocyclic groups as "H-groups." An H-group, then, is a group Γ of linear transformations of a complex variable such that

(a) Γ is discontinuous in the upper half-plane but is not discontinuous at any point of the real axis,

(b) every transformation of Γ preserves the upper half-plane,

(c) Γ contains parabolic substitutions with fixed point ∞.

The main object of this paper is to determine, by the use of the circle method, the expansions of the Fourier coefficients of automorphic forms on H-groups of a certain class. The circle method has been employed by Rademacher and Zuckerman ([6], [7], [10], [11]) for the case where the H-group is the modular group or one of its subgroups. Here we develop the circle method for a class of H-groups defined by the following

(1.0) RESTRICTION: *A fundamental region of the* H-*group shall have exactly one parabolic cusp.*

(This implies that the fundamental region has a finite number of sides ([2], Thm. 16, p. 75).) As a consequence of this condition, there exists a number $h > 0$ such that the fundamental region with cusp at ∞ does not extend below the horizontal line at height h above the real axis.

The circle method is elementary in character; it uses only Cauchy's theorem and a careful dissection of the path of integration. Lacking an arithmetic characterization of the parabolic points of the H-group such as is available in the case of the modular group, we use the geometry of the fundamental region for the dissection of the path.

We treat entire automorphic forms of dimension r, that is, analytic functions of a complex variable z which are regular in the upper half-plane and satisfy there the functional equation

$$F(Vz) = \varepsilon(V)(-i(cz + d))^{-r} F(z), \tag{1.1}$$

for every transformation $Vz = (az + b)/(cz + d)$ of Γ. Here r is real, and the multiplier $\varepsilon(V)$ is independent of z and satisfies the condition $|\varepsilon(V)| = 1$. If $c \neq 0$, we assume that $c > 0$ and require $\arg(-i(cz + d))$ to lie between $-\pi/2$ and $\pi/2$, as a means of determining the branch of the many-valued function. If $c = 0$, we have, as we shall presently see, $Vz = z + m\lambda$, where m is an integer and $\lambda > 0$. Set $Sz = z + \lambda$ and

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