ON THE IMPOSSIBILITY OF FIBRING CERTAIN MANIFOLDS BY A COMPACT FIBRE

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1. INTRODUCTION

By a proper fibration of a space we shall understand a fibration in which the fibre is not a single point. It is well known that a Euclidean space does not admit a proper fibration by a compact fibre [2], [6]. It is the purpose of this note to extend this result to a wider class of spaces.

THEOREM 1. *Let* W^n *be an open, simply connected manifold whose one-point compactification is again a manifold; then* W^n *does not admit a proper fibration by a compact fibre.*

To see that a hypothesis implying orientability is needed, we observe that the manifold obtained by removing a point from the real projective plane admits the cyclic group Z_p, for every odd prime, as a fixed-point free group of transformations.

To prove the theorem, we first examine the case in which the fibre is connected. By means of a theorem of Spanier and Whitehead [7], we show that the fibre is an H-space. With the help of Borel's principal algebraic theorem in [1], we show further that the fibre is a rational cohomology sphere. An argument based on the Gysin sequence shows that this is impossible. We are then reduced to showing that a finite group cannot act freely on W^n, and this follows from a result of Mostow, which, incidentally, suggested our note.

2. PRELIMINARIES

A space W^n whose one-point compactification is a manifold is said to be locally Euclidean at infinity. Given such a W^n, for every open set $U \subset W^n$ with $\overline{U}$ compact, there is an open set V such that $\overline{U} \subset V$, $\overline{V}$ is compact, and $W^n - V$ is homeomorphic to a closed n-cell with the origin removed.

We shall denote by M^n the compact manifold obtained by adjoining the point at infinity to W^n, and by $p \in M^n$, the added point. By $[W^n, B, F; \pi]$ we denote a proper fibration of W^n over the base space B with compact fibre F and projection map π. By the term *fibration* we shall mean local product structure. Given $[W^n, B, F; \pi]$, we denote by $\hat{B}$ the one-point compactification of B, by $g \in \hat{B}$ the added point, and by $\hat{\pi}: M^n \to \hat{B}$ a map for which

$$\hat{\pi} \mid M^n - p = \pi, \qquad \pi(p) = g.$$

It will be useful to regard $[M^n, p, \hat{B}, g, F; \hat{\pi}]$ as a singular fibration [5].

It is immediately seen from the local product structure that in a fibration of a manifold, such as $[W^n, B, F; \pi]$, both the fibre and the base are well behaved with

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