ON THE ACTION OF THE CIRCLE GROUP

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1. INTRODUCTION

The object of this note is to complement a recent series of discussions ([4], [5], [6]) of the operation of the circle group as a group of transformations. We shall consider transformation groups of the form (S^1, X), where X is locally compact, locally connected and separable metric. Associated with each (S^1, X) are two spaces in whose rational cohomology groups we shall be interested: the set $F \subset X$ of stationary points of (S^1, X), and the orbit space X/S^1. Smith ([2], [9]) has established a number of theorems about the operation of the integers mod p, Zp, as a group of transformations. These theorems concern the cohomology structure of the associated spaces, and the coefficients Zp are used. We shall establish the analogous theorems about the rational cohomology structure under an action of the circle group. Actually, by making use of the cohomology ring structure in our arguments, we are able in some places to avoid finite dimensionality restrictions on X. We are principally interested in illustrating techniques which may prove of value in later studies of compact transformation groups.

2. PRELIMINARIES

We denote by $H^i(X; Q)$ the rational Alexander-Wallace-Spanier cohomology groups of X, and by $H^i_c(X; Q)$ those cohomology groups which are based on cochains with compact support ([3]). Most of our arguments employ spectral sequences, and we shall assume familiarity with relevant techniques; for more details, we refer to ([1], [3]).

We recall that there is a canonical transformation group of the form (S^1, S^{2i+1}) such that S^{2i+1}/S^1 is the complex projective space $P(i)$. Indeed, there is a whole sequence of such operations and a natural equivariant imbedding making an ascending sequence,

$$p_i\colon (S^1, S^{2i+1}) \to (S^i, S^{2(i+1)+1});$$

for if we form $(S^1, \bigcup_i S^{2i+1})$, we obtain the universal bundle for S^1, where $\bigcup_i S^{2i+1}$ is given the CW-topology ([1, p. 165]). We say that a transformation group (S^1, X) is *almost free* provided every isotropy subgroup is finite. For every almost free transformation group we form $(S^1, Y(i))$, where $Y(i) = S^{2i+1} \times X$ and S^1 acts on this cartesian product by diagonal action. Consider the natural diagram

$$\begin{array}{ccc} & (S^{2i+1} \times X)/S^1 & \\ \alpha \swarrow & & \searrow \beta \\ P(i) & & X/S^1 \end{array}$$

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