LOCALLY PROJECTIVE SPACES OF DIMENSION ONE

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Let P be the real projective line, and let $\tilde{P}$ be the universal covering space of P with the lifted projective structure. Topologically, $\tilde{P}$ is an interval. Every projective mapping of a neighborhood in $\tilde{P}$ onto another neighborhood is the restriction of a unique projective homeomorphism of $\tilde{P}$ onto $\tilde{P}$. A projective transformation of P can be expressed with respect to homogeneous or nonhomogeneous preferred coordinates as follows:

$$\text{homogeneous coordinates: } (X, Y) \to (X^*, Y^*) = (aX + bY, cX + dY), \; ad - bc \neq 0;$$

$$\text{nonhomogeneous coordinates: } (X = xY) \; x \to x^* = \frac{ax + b}{cx + d}.$$

A *locally projective space* Z of dimension 1 is a manifold and a complete (that is, not properly contained in a larger) *atlas* of mutually compatible homeomorphisms, called *maps*, of neighborhoods in Z onto neighborhoods in $\tilde{P}$. Two maps $f: U \to U'$ and $g: V \to V'$ are *compatible* if, for each connected component W of the intersection $U \cap V$, the mapping gf^{-1} restricted to $f(W)$ is a projective transformation in $\tilde{P}$.

Two maps in the atlas, both covering the point z in Z, are said to be equivalent at z if their restrictions to some neighborhood of z coincide. Following Ehresmann [2], we call an equivalence class of maps in the atlas, all covering z, *a local jet*, and we denote it by j_z. A topology in the set of jets can be introduced by giving a base for the open sets as follows: the set of jets (that is, of equivalence classes of maps) which contain a map is an open set in the space of jets.

A connected component of the space of jets is a covering space of Z, with the projection $j: j_z \to z$. The mapping which sends a jet j_z of this covering space into the image in $\tilde{P}$ of the point z under each of the maps of the equivalence class j_z is a homeomorphism onto an interval in $\tilde{P}$. This interval can therefore be considered as the universal covering space $\tilde{Z}$ with lifted locally projective structure of Z.

An analogous theorem and proof, given in [2], exist in the case of the locally homogeneous spaces, where the homogeneous space with a transitive Lie group of transformations takes the place of the projective line in the present article (see [1] to [9]).

There exist only two manifolds of dimension 1, in the topological sense: the open interval and the circle. We consider them separately.

Case A. Z *is topologically an interval*. Here $\tilde{Z}$ is projectively equivalent with an interval in $\tilde{P}$. The classification into projectively different cases is as follows:

A1. $Z = \tilde{P}$.

A2. Z is one of the two parts into which a point in $\tilde{P}$ divides its complement in $\tilde{P}$.

Received by the editors in March, 1954.