NOTE ON MY PAPER "INTRINSIC RELATIONS SATISFIED BY THE VORTICITY AND VELOCITY VECTORS IN FLUID FLOW THEORY"

by

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The purpose of this note is: (1) to relate the papers of several other authors to the above paper[1] of the present author; (2) to provide some further details for the derivation of one equation of the above paper and to generalize a theorem of S. S. Byušgens; (3) to indicate extensions[2] of our previous results to more general types of gases.[3]

1. Related Papers by other Authors. Our previous equation (2.19) for the decomposition of the vorticity vector is equivalent to the relation 2.3 (6) of Bjørgum's paper.[4] This can be seen with the aid of the following computation. If e_{ijk}, e^{ijk} denote the permutation tensor in an orthogonal Cartesian coordinate system, x^j, $j = 1, 2, 3$, then it is well known that

$$e^{ijk}e_{imp} = \delta^j_m \, \delta^k_p - \delta^j_p \, \delta^k_m \tag{1.1}$$

where δ^j_n is the Kronecker tensor. From (1.1), it follows that

$$(b^j n^k - b^j n^k)\, \partial_j t_k = (e_{imp} b^m n^p)(e^{ijk}\, \partial_j t_k) . \tag{1.2}$$

The left hand side of (1.2) can be written in form

$$n^k \frac{\partial t_k}{\partial b} - b^k \frac{\partial t_k}{\partial n} ,$$

Further, the first term of the right hand side of (1.2) is the cross-product of the unit vectors, $\tilde{b}$ and $\tilde{n}$, or the negative of the unit tangent vector, $\tilde{t}$; similarly, the second term of the right hand side of (1.2) is the curl of $\tilde{t}$. Thus, (1.2) reduces to

$$b^k \frac{\partial t_k}{\partial n} - n^k \frac{\partial t_k}{\partial b} = \tilde{t} \cdot \text{curl } \tilde{t} . \tag{1.3}$$

The relation (1.3) and 2.3 (6) of Bjørgum's paper lead to our relation (2.19). Similarly, our equation (2.21), which is a generalization of (2.19), is equivalent to Bjørgum's equation 2.6 (17). In fact, Bjørgum's dyadic decomposition of Section 2.6 and our tensor decomposition of Section 2 are closely related.

Our previous equation (4.16) results from applying the divergence to the vector relation.

$$\tilde{v} = q\, \tilde{t} \tag{1.4}$$

and showing that

$$\text{div. } \tilde{t} = h_1 + h_2 \tag{1.5}$$

where h_1, h_2 are the two principal values of the symmetric part of the projection of the tensor, grad $\tilde{t}$, in the plane locally perpendicular to $\tilde{t}$. When

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