A JACOBIAN CONDITION FOR INTERIORITY

by C. J. Titus and G. S. Young

Let D be an open set in Euclidean n-space, E^n, and let f: $D \to E^n$ be a continuous function from D into E^n. There are a number of topological conditions on f so that f is interior on D; that is, so that the image of every open subset of D is open in E^n. However, these conditions are not formulated in such a way as to yield simple proofs of interiority for functions as they occur naturally in analysis. For example, a function analytic in a plane domain is interior, but the writers know of no proof of interiority that does not employ such tools as Taylor series or the integration theory of analytic functions. (For a discussion of this case, see Whyburn's Memoir [4].) In this note, we provide simple sets of conditions for interiority and for quasi-interiority.

Definitions. A mapping f: $A \to B$ is _light_ if for each point b of B, $f^{-1}(b)$ is totally disconnected. It is _monotone_ if for each point b of B, $f^{-1}(b)$ is connected. It is _quasi-interior_ (Whyburn) if for each point b is B and for each open set U in A that contains a compact component of $f^{-1}(b)$, b is in Int f(U), the interior of f(U). Clearly a light quasi-interior transformation is interior.

Theorem 1. Let D be an open set in E^n. Let f: $D \to E^n$ be of class C^1, and let the Jacobian J(f(p)) be zero only on a compact subset of D of dimension less than n − 1. Then f is quasi-interior on D.

Proof. If Z denotes the subset of D on which