A COMBINATORIAL PROBLEM [1]

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1. Introduction.

Let m be a natural number and let (a) = ($a_1, \ldots, a_k$), $a_1 > a_2 > \ldots > a_k > 0$, be a partition of m into unequal parts. We associate with the partition (a) a diagram D(a) of m nodes (or places) having a_1 rows and k columns, so arranged that the j-th column has a_j nodes, and that the top node of the j-th column is in the j-th row (numbered from the top down).
For example,

D(5,3,2)=
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. . .
. . . .
.
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and D(4,3,2,1)=
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. .
. . .
. . . .
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.

A one-to-one mapping of the nodes of D(a) onto the set of natural numbers 1, ..., m is called a <u>labelling</u> of D(a) and is indicated by writing in each place of D(a) its image under the mapping. A labelling is said be be <u>regular</u> if the "labels" increase in each row when read from left to right and increase in each column when read from top to bottom. Thus the first of the two labellings below is regular whereas the second is not:

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1                    1
2 4                  3 2
3 5 7        ,       4 7 10      ,
6 8 9 10             6 5 8 9
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We denote by g(a) the number of regular labellings of D(a). The main result in this note is a formula for g(a). (See Theorem 1). The problem of

[1] The work on this paper was performed under the sponsorhip of the ONR.