ON THE SUMMABILITY OF ORDINARY DIRICHLET SERIES BY TAYLOR METHODS

by

V.F. Cowling and G. Piranian

For any real constant α in the interval $0 \le \alpha \le 1$, the symbol T_α shall denote the regular sequence-to-sequence transformation represented by the upper-triangular Toeplitz matrix $(t_{n,k})$, where

$$t_{nk} = (1-\alpha)^{n+1} C_{k,n} \alpha^{k-n}$$

for $n = 0, 1, \ldots$ and $k \ge n$, and $t_{nk} = 0$ for $k < n$. The transformations T_α were introduced as "circle methods" by G.H. Hardy and J.E. Littlewood [2] in connection with a certain Tauberian theorem on the Borel transformation. R. Wais [6] and W. Meyer-König [3] made extensive investigations concerning the application of these transformations to Taylor series, and they introduced the name Taylor-Verfahren. The transformations T_α were again introduced, independently and without the restriction of α to real values, by P. Vermes [4], [5], and by V. F. Cowling [1].

In the present paper we prove two theorems concerning Taylor transformations of ordinary Dirichlet series. It is convenient to replace the transformations T_α by the corresponding series-to-series transformations V_α :

$$V_\alpha \sum_{n=0}^{\infty} a_n = \sum_{n=0}^{\infty} b_n ,$$

where

$$b_n = \sum_{k=n}^{\infty} v_{nk} a_k \text{ and } v_{nk} = (1-\alpha)^n C_{k,n} \alpha^{k-n}$$

for $k \ge n$, $v_{n,k} = 0$ for $k < n$. If the V_α transform of