A SIMPLE PROOF OF THE ERDÖS-MORDELL INEQUALITY FOR TRIANGLES

Donat K. Kazarinoff
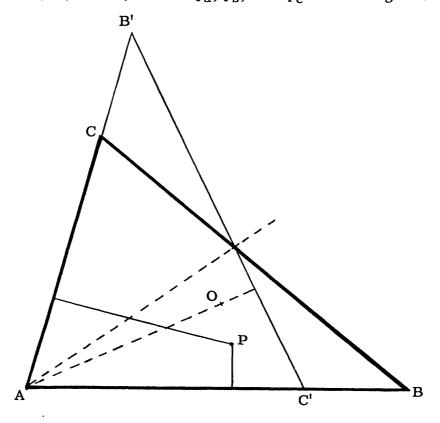
THEOREM (Erdös-Mordell). *For any triangle and any point* P *interior to it or on its boundary, the sum* S_1 *of the distances from* P *to the sides and the sum* S_2 *of the distances from* P *to the vertices satisfy the inequality*

$$S_2 \geq 2S_1;$$

equality holds only in the case where the triangle is equilateral and P *is its centroid.*

For a discussion of this theorem, see [2, pp. 12-14, 28]. My proof of it is based on the following theorem of Pappus [3; Book 4, Proposition 1]: Let ABC be any triangle, and let ABDE and ACFG be two parallelograms of which either both or neither lies entirely outside of ABC. Let H be the point of intersection of DE and FG (extended), and let BCKL be a parallelogram whose side CK is a translate of the vector AH. Then the sum of the areas of ABDE and ACFG is equal to the area of BCKL. I also use the well-known fact (see [1, p. 53]) that the bisector of any angle of a triangle is also the bisector of the angle made by the diameter of the circumcircle and the altitude drawn through the vertex.

Let a, b, and c be the lengths of the sides of the triangle ABC opposite the respective vertices A, B, and C, and let p_a, p_b, and p_c be the lengths of the

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