The Intersection of a Linear Subspace with the Positive Orthant

by

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1. This note discusses the geometry of those convex polyhedral cones[1] in Euclidean n-space E^n which are the intersection of the positive orthant P^n with some linear subspace. Cones of this sort occur in linear programming problems (cf. e.g. G. B. Dantzig, AAPA, p. 360). However, the present work was motivated by its application to a geometrical description of the frame of an arbitrary convex polyhedral cone, which description will appear in a subsequent paper.

Terminology and notation used in this paper will be defined only where they depart from those of Gerstenhaber (AAPA, Chap. XVIII).

2. In any Euclidean space E^m, the (closed) positive orthant P^m is the set of all vectors a such that $a \geqq 0$.

Given any $n \times k$ matrix A, that is, any linear transformation on E^k to E^n, the image AP^k of P^k under A is evidently the set of all positive linear combinations of the columns of A, considered as vectors in E^n. The class of all AP^k, for all k and A, is therefore by definition exactly the class of all convex polyhedral cones.

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1. See Cowles Commission, Activity analysis of production and allocation, ed. T. C. Koopmans (cited hereafter as AAPA), particularly Chaps. SVII (by D. Gale) and XVIII (by M. Gerstenhaber).