Intrinsic Relations Satisfied by the Vorticity and Velocity Vectors in Fluid Flow Theory

by

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1. Introduction. In plane fluid flows, it is well known[1] that the following relation exists between the magnitude of the vorticity vector, ω, and that of the velocity vector, q,

$$\omega = \frac{-\partial q}{\partial n} + K q . \tag{1.1}$$

Here, K is the curvature of the stream line at the point under consideration, and $\partial q/\partial n$ represents the rate of change of q with respect to arc length along the direction normal to the stream line.

Our first problem is to generalize the above relation to three dimensional fluid flow theory. To do so, we shall decom pose the vorticity vector into components along the tangent, principal normal, and bi - normal to the stream lines. From this decomposition, the desired generalization of formula (1.1) is easily obtained. It will be shown that the right hand side of (1.1) is nothing but the component of the vorticity vector along the binormal direction.

Secondly, we shall determine an intrinsic relation satisfied by the Bernoulli function for the case of the steady flow of a non-viscous fluid. From this relation, one can easily obtain a necessary and suf-

1) Theoretical Hydrodynamics, L. M. Milne - Thomson, Macmillan Co, London, 1938, p. 99.