TERNARY RELATIONS IN GEOMETRY AND ALGEBRA

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One of the simplest sets of axioms used in Mathematics is the set of propositions defining an equivalence relation. We will express the assertion that a is equivalent to b by writing (ab)'. For our present purposes it is convenient to state that this binary relation possesses the properties (the symbol > means "implies" in what follows):

(Se) Symmetry, that is, (ab)' > (ba)'
(Te) Transitivity, that is, (ab)', (bc)' > (ac)'.

The starting point of the following discussion was the observation that in many places in Mathematics similar although more complicated propositions occur, or that it is often possible to reformulate the discussion so that it can be stated in similar terms. An example of a ternary relation is that of collinearity. If we denote the fact that the points A, B, C lie on a straight line by writting (ABC)* we can say that this relation possesses the property

(Sg) Symmetry, that is (ABC)* > (BCA)*, (BAC)* etc.
and the property

(Tg) If A and B are distinct points

$$(ABC)^*, \ (ABD)^* > (ACD)^*.$$

Clearly, the last property is of the same general type as the transitivity property of the equivalence relation; the difference is essentially that there we deal with a binary relation and three elements and here, in the case of collinearity, with a ternary re-