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Automorphism Groups and Derivation Algebras of Finitely Generated Vertex Operator Algebras

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1. Introduction

In this paper we investigate the general structure of the automorphism group and the Lie algebra of derivations of a finitely generated vertex operator algebra. We prove two main results. The automorphism group is isomorphic to an algebraic group. Under natural assumptions, the derivation algebra has an invariant bilinear form and the ideal of inner derivations is nonsingular.

Definition 1.1. Let V be a vertex operator algebra. We say that $a \in \mathrm{GL}(V)$ is an *automorphism of* V if and only if it leaves the vacuum element and the principal Virasoro element fixed ($a\mathbf{1} = \mathbf{1}$ and $a\omega = \omega$) and preserves all V-compositions; that is, for all $m \in \mathbb{Z}$ and $u, v \in V$, we have $a(u_m v) = a(u)_m a(v)$. It follows that an automorphism fixes all the V_i since they are eigenspaces for an operator in the series for the principal Virasoro element.

The set of all automorphisms is a group, denoted $\mathrm{Aut}(V)$.

In the definition, it suffices to restrict u and v to homogeneous elements. Note that, in some definitions of VOA automorphism, there is no requirement that the principal Virasoro element be fixed.

So far, we know the automorphism groups explicitly for relatively few vertex operator algebras, such as $V^\natural$ [FLM], vertex operator algebra V_L for a positive definite even lattice L [DN], certain vertex operator algebras with central charge 1 [DG; DGR], vertex operator algebras associated to highest weight representations for affine algebras (cf. [DLY]), vertex operator algebras associated to codes [M], and a few special cases (see e.g. [G]).

The determination of each of these automorphism groups has its own story and depends heavily on the specifics of the auxiliary object used to construct the VOA, such as a lattice, Lie algebra, or code. Nevertheless, one can observe that all these automorphism groups have similarities.

We denote by (V, k^{th}) the algebra with underlying vector space V and product $a_k b$ for $a, b \in V$, where a_k is the coefficient at z^{-k-1} in the vertex operator for a.

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