Michigan Math. J. 49 (2001)

On the Analyticity of Smooth CR Mappings between Real-Analytic CR Manifolds

Sylvain Damour

Introduction

Let $M \subset \mathbb{C}^n$ ($n \geq 2$) be a generic real-analytic CR submanifold, $M' \subset \mathbb{C}^{n'}$ a real-analytic subset, and $f : M \to M'$ a smooth (i.e., of class $\mathcal{C}^\infty$) CR mapping defined near the point $p \in M$. It is natural to ask the following question: Under what conditions is f real-analytic? (Of course, in this case, it extends holomorphically to a neighborhood of p.)

In the equidimensional case, many authors considered the situation when f is a CR diffeomorphism [3; 16; 18; 20; 23; 27]. The more general situation, when f is supposed to be of only finite multiplicity, was studied in [4; 17]. When M and M' have different dimensions, more recent results give also some sufficient conditions [12; 13; 19]. We would also like to mention related works on the regularity of continuous CR mappings [10; 11; 14; 28].

In this paper, generalizing the result of Coupet, Pinchuk, and Sukhov [12] to arbitrary codimension, we give a new sufficient condition for the analyticity of a smooth CR mapping $f : M \to M'$ between a generic real-analytic submanifold $M \subset \mathbb{C}^n$ and a real-analytic subset $M' \subset \mathbb{C}^{n'}$. We prove that, if M is minimal at $p \in M$ and if the characteristic variety of f at p is 0-dimensional, then f is real-analytic near p (see Theorem 1.2). Our result generalizes many situations previously considered by other authors:

(1) $M, M' \subset \mathbb{C}^n$ are hypersurfaces, M' is strictly pseudoconvex, and f is a CR diffeomorphism (Lewy [23] and Pinchuk [27]);
(2) $M, M' \subset \mathbb{C}^n$ are submanifolds, M is minimal, M' is essentially finite, and f is a CR diffeomorphism (Baouendi, Jacobowitz, and Trèves [3]);
(3) $M, M' \subset \mathbb{C}^n$ are hypersurfaces, M' is essentially finite, and f is of finite multiplicity (Diederich and Fornæss [17] and Baouendi and Rothschild [4]).

We point out that our main result applies to situations not listed here—in particular, when M and M' have different dimensions. Our main result seems to be new also in the equidimensional case, when $M, M' \subset \mathbb{C}^n$ are submanifolds of *higher codimension*. In this case, our sufficient condition can be seen as a generalization of the finite multiplicity condition of [4; 17].

In this paper, we introduce the notion of "characteristic variety" associated to the sets M and M' *and* to the mapping f; this is a generalization to higher codimension

Received January 30, 2001. Revision received April 16, 2001.