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Valuative Arf Characteristic of Singularities

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1. Introduction

The proof by Hironaka [5] of resolution of singularities of algebraic varieties over fields of characteristic zero raised the problem of classifying singularities by looking at the resolution process. Thus, equisingularity of plane curve singularities was introduced and developed by Zariski in [15] by showing that the combinatorics of the resolution processes is equivalent data to Puiseux invariants, that is, the numerical data consisting of the Puiseux exponents of the branches and the intersection multiplicities among pairs of branches. For space curves, it is known that the combinatorics of the resolution processes is equivalent data to the Arf characteristic (see [5]). Arf closure lies between the singularity and its normalization, and its definition can be given in terms of the set of valuations centered at the singularity. Notice that, since those valuations correspond one-to-one to branches of the curve, the aforesaid set is finite and is canonically associated to the singularity.

In higher dimension the situation becomes much more complicated, as the resolution of singularities is not unique at all. In this paper we define the Arf characteristic for schemes of arbitrary dimensions. We define Arf closure relative to a finite set of valuations centered at a singularity and study its algebraic-geometric properties. Using appropiate canonically defined sets of divisorial valuations, we define the *valuative Arf characteristic of singularity* (note that it is not related at all to the well-known *Arf invariants* of the $\mathbb{Z}_2$ quadratic form associated topologically with the link of plane curve singularities). These invariants can be viewed as a generalization of Puiseux characteristic to higher dimensions. Arf closure relative to a single divisorial valuation was introduced in [2], showing that the corresponding invariants describe the geometry of certain arcs on the singularity.

There are two natural sets of valuations canonically associated to a singularity. First, one can associate the so-called essential valuations: those valuations that appear explicitly at every resolution. For surface singularities one has a minimal resolution, so that the essential valuations are nothing but the divisorial valuations centered at the components of the exceptional divisor of the minimal resolution. For dimension higher than two, essential valuations are not determined except in a few cases (see e.g. [1]). A result by Nash of 1964 (recently published in [10]) shows that the set of essential valuations contains the set of valuations coming

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