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Smooth Structure of Some Symplectic Surfaces

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1. Introduction

McMullen and Taubes [MT] have constructed a remarkable simply connected smooth 4-manifold, denoted by X, starting from a 4-component link $K \subset S^3$ and four copies of the rational elliptic surface $E(1)$. The interest in the link K stems from the fact that it admits several inequivalent fibrations over S^1; these inequivalent fibrations give rise to two inequivalent symplectic structures on X, providing the first simply connected example of manifold with this property. The ingredients in the construction of [MT] are reminiscent of those used by Fintushel and Stern in defining a large class of smooth 4-manifolds, and it is natural to ask how these constructions are related. In this note we will compare the link surgery construction of [FS] and the McMullen–Taubes example in order to prove that the latter manifold is diffeomorphic to a Fintushel–Stern manifold. This analysis (further developed in [V]) will lead us to introduce a new presentation of X that allows us to identify a new symplectic structure on X. We will assume some familiarity with [FS] and [MT].

2. Construction of the 4-Manifolds

We start by recalling the link surgery construction of [FS], omitting (for the sake of brevity) full generality. Consider an n-component oriented link $K \subset S^3$. Let $p_i = -\sum_{j \neq i} lk(K_i, K_j)$. The closed manifold M_K obtained by performing p_i-surgery on the ith component has the property that the image m_i of each meridian $\mu(K_i)$ has infinite order in $H_1(M_K, \mathbb{Z})$ and is canonically framed; in $S^1 \times M_K$, the tori $S^1 \times m_i$ have self-intersection zero and are framed and essential in homology. Next take n copies of the simply connected elliptic surface without multiple fibers $E(m)$, each containing an elliptic fiber F_i, and construct, by normal connected sum, the manifold

$$E(m)_K = \coprod E(m)_i \#_{F_i = S^1 \times m_i} S^1 \times M_K. \tag{1}$$

The gluing is made so as to send the homology class of the normal circle to the ith torus $S^1 \times m_i$, represented by $p_i m_i + l_i$ (where l_i is the image of the preferred longitude $\lambda(K_i)$) to the class of a normal circle to the ith elliptic fiber. These

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