Michigan Math. J. 49 (2001)

Lipschitz Estimates for the $\bar{\partial}$-Equation on the Minimal Ball

N. Viêt Anh & E. H. Youssfi

1. Introduction and Statement of the Main Results

The general theory of the $\bar{\partial}$-equation on convex domains in $\mathbb{C}^n$ is still incomplete. It has been studied in several particular cases of smooth convex domains; see, for example, the articles of Range [21], Diederich, Fornæss, and Wiegerinck [7], Bruna and Castillo [3], Bonami and Charpentier [2], Cumenge [5], and Diederich, Fischer, and Fornæss [6]. In these works, the regularity estimates for the $\bar{\partial}$-equation depend intimately on the geometry of the boundary of the domain. For example, if the domain is smooth convex of finite type m then the sharp gain of smoothness is $1/m$ (see [5; 6]). In proving these results, the boundary smoothness is used heavily. The particular case of smooth strictly pseudoconvex domains corresponds to the $\frac{1}{2}$-regularity. This smoothness has been shown to hold even in the case of nonsmooth strictly pseudoconvex domains with, however, a $\mathcal{C}^2$-defining function (see Henkin and Leiterer [12]).

On other hand, Fornæss and Sibony [8] constructed a smoothly bounded pseudoconvex domain that is strictly pseudoconvex except at one boundary point for which (L^p, L^p)-estimates $(p > 2)$ for $\bar{\partial}$ fail.

In the present work we give an example of a convex circular and non–piecewise smooth domain, with a defining function that is not differentiable, for which the $\bar{\partial}$-equation possesses the Lipschitz $\widetilde{\frac{1}{2}}$-estimate. We also give an explicit construction of the $\bar{\partial}$-solving operator. The domain in question is the minimal ball, which is given by

$$\mathbb{B}_* := \{z \in \mathbb{C}^n : \varrho(z) := |z|^2 + |z \bullet z| < 1\},$$

where $z \bullet w := \sum_{j=1}^n z_j w_j$ (see Hahn and Pflug [10]). Then the minimal ball $\mathbb{B}_*$ is just the open unit ball with respect to the norm $N_* := \sqrt{\varrho}$, as featured in several recent works [13; 15; 16; 17; 18; 19; 20; 24; 25]. In particular, it is a non–Lu Qi-Keng domain for $n \geq 4$ and is neither homogeneous nor Reinhardt. In addition, $\mathbb{B}_*$ has a B-regular boundary in the sense of Sibony [23] and Henkin and Iordan [11].

Set $V := \{z \in \mathbb{C}^n \setminus \{0\} : z \bullet z = 0\}$. The singular part of the boundary of $\mathbb{B}_*$ is obviously the set $\partial\mathbb{B}_* \cap V$. The regular part $\partial\mathbb{B}_* \setminus V$ consists of all strictly pseudoconvex points.

Received September 7, 2000. Revision received January 29, 2001.