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Triangular Hopf Algebras with the Chevalley Property

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1. Introduction

Triangular Hopf algebras were introduced by Drinfeld [Dr]. They are the Hopf algebras whose representations form a symmetric tensor category. In that sense, they are the class of Hopf algebras closest to group algebras. The structure of triangular Hopf algebras is far from trivial and yet is more tractable than that of general Hopf algebras, owing to their proximity to groups and Lie algebras. This makes triangular Hopf algebras an excellent testing ground for general Hopf algebraic ideas, methods, and conjectures.

A general classification of triangular Hopf algebras is not known yet. However, there are two classes that are relatively well understood. One of them is semisimple triangular Hopf algebras over **C**, for which a complete classification is given in [EG1; EG2]. The key theorem about such Hopf algebras states that each of them is obtained by twisting a group algebra of a finite group (see [EG1, Thm. 2.1]). The proof of this theorem is based on Deligne's theorem on Tannakian categories [D1].

Another important class of Hopf algebras is that of *pointed* ones. These are Hopf algebras whose simple co-modules are all 1-dimensional. Theorem 5.1 in [G] gives a classification of minimal triangular pointed Hopf algebras (we note that the additional assumption made in [G, Thm. 5.1] is, by our Theorem 6.1, superfluous).

Recall that a finite-dimensional algebra is called *basic* if all of its simple modules are 1-dimensional (i.e., if its dual is a pointed co-algebra). The same Theorem 5.1 of [G] gives a classification of minimal triangular basic Hopf algebras, since the dual of a minimal triangular Hopf algebra is again minimal triangular.

Basic and semisimple Hopf algebras share a common property—namely, the Jacobson radical $\mathrm{Rad}(H)$ of such a Hopf algebra H is a Hopf ideal and therefore the quotient $H/\mathrm{Rad}(H)$ (the semisimple part) is itself a Hopf algebra. The representation-theoretic formulation of this property is: The tensor product of two simple H-modules is semisimple. A remarkable classical theorem of Chevalley [C, p. 88] states that, over **C**, this property holds for the group algebra of any (not necessarily finite) group. So let us call this property of H the *Chevalley property*.

The Chevalley property certainly fails for many finite-dimensional Hopf algebras—for example, for Lusztig's [L] finite-dimensional quantum groups $\mathrm{U}_q(\mathbf{g})'$

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