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Parabolic Manifolds for Semi-Attractive Holomorphic Germs

MARZIA RIVI

1. Introduction

The purpose of this paper is to study the local behavior of *semi-attractive* holomorphic self-maps of $\mathbb{C}^m$ $(m > 2)$ in a neighborhood of a fixed point that we assume to be the origin. Such transformations are the ones whose differential at 0 has one eigenvalue equal to 1 while the remaining ones, say $\beta_1, \ldots, \beta_s$ with $s \geq 1$, have modulus strictly less than 1.

Semi-attractive transformations such that 0 is not an isolated fixed point have been studied by Nishimura [N], who considered analytic automorphisms F of complex manifolds admitting a q-dimensional complex submanifold M of attracting fixed points for F. Then, for each point $p_0 \in M$, one can choose local coordinates $(w, z) \in \mathbb{C}^q \times \mathbb{C}^{m-q}$ in a neighborhood U of p_0 such that $U \cap M$ has equation $z = 0$; hence the map F can be locally written as

$$\begin{aligned} w_1 &= w + O(\|z\|), \\ z_1 &= C(w)z + O(\|z\|^2), \end{aligned}$$

where $C(w)$ is a $(m-q) \times (m-q)$ matrix whose elements are holomorphic functions on U and whose eigenvalues $\beta_1(w), \ldots, \beta_{m-q}(w)$ have modulus strictly less than 1.

Let $\Omega = \{p \in U \mid F^n(p) \to p_0,\ p_0 \in M\}$, which is an open set containing M. Nishimura proved that if these eigenvalues have no relations in any point of M, that is, if for each multi-index $j = (j_1, \ldots, j_{m-q})$ with $\sum_{k=1}^{m-q} j_k \geq 2$ and $1 \leq i \leq m-q$ we have $\beta_1^{j_1}(w) \cdots \beta_{m-q}^{j_{m-q}}(w) \neq \beta_i(w)$, then there exists a biholomorphism $S: N \to \Omega$, where $\pi: N \to M$ is the normal bundle of M, which conjugates F to the automorphism G of N induced by F and given in $\pi^{-1}(U \cap M)$ by

$$\begin{aligned} s_1 &= s, \\ u_1 &= C(s)u. \end{aligned}$$

When 0 is an isolated fixed point, the problem has been studied only for maps such that the eigenvalue 1 has multiplicity $q = 1$. Fatou [F] and subsequently Ueda [U1; U2] studied the dynamics of such transformations F in two complex variables. Fatou found a system of coordinates (w, z) such that F has the following expression in a neighborhood of 0:

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