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AN EXTENSION OF VENN DIAGRAMS

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Part One: World-state Diagrams

In *Methods of Logic* Quine mentions two limitations of Venn-Diagrammatic techniques. The first is the well-known difficulty of constructing a diagram for a large number of terms. But Venn himself suggested a way to get around this difficulty, viz. to renounce all hope of generating a k-term diagram by superimposing k simple closed curves and instead to subdivide a rectangle into the requisite number of sub-compartments or bins, i.e. 2^k of them.[1] Marquand's rectangular graphs seem simply to incorporate this suggestion. Despite the fact that Marquand-graphs (hereafter **M**-graphs) are readily available for any finite number of terms, they seem to be no more capable than Venn-diagrams of representing arguments which involve, in Quine's phraseology, "an admixture of truth functions"[2] and which present "another place where the unaided method of diagrams bogs down".[3] Quine cites the following as an example of an argument form involving an admixture of truth functions:

(*All FG are H*) $\supset$ (*Some F are not G*)
(*All F are G*) $\vee$ (*All F are H*)
Thus, (*All FH are G*) $\supset$ (*Some F which are not H are G*).[4]

Referring to this example Quine rhetorically asks "just how may we splice the two techniques in order to handle a combined inference of the above kind?"[5]. (The two techniques mentioned are Venn-diagrams and truth-value analysis.) This paper is an answer to Quine's rhetorical question. It shows how to splice Venn-diagrammatic and truth-tabular techniques so as to get a *diagrammatic* decision procedure applicable to all arguments of the above kind, i.e. to uniform quantification theory.[6] If, furthermore, one appends to it some simple but non-truth-functional transformations, the decision procedure becomes applicable to the whole of monadic quantification theory.[7] In addition these diagrams, which will be called *world-state diagrams* (hereafter **WSD**s), provide an intuitive basis on which to define the notions of validity and semantical completeness of both uniform and monadic quantification theory.

Before introducing **WSD**s, I wish to point out a little-noticed fact, viz.

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