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THE SUBSTITUTION SCHEMA IN RECURSIVE ARITHMETIC

R. D. LEE

In his paper *Logic Free Formalisations of Recursive Arithmetic* [1] R. L. Goodstein presents a formalisation of primitive recursive arithmetic in which the only axioms are explicit and recursive function definitions, and the rules of inference are the schemata

(Sb₁) $$\frac{F(x) = G(x)}{F(A) = G(A)}$$

(Sb₂) $$\frac{A = B}{F(A) = F(B)}$$

(T) $$\frac{\begin{array}{c} A = B \\ A = C \end{array}}{B = C}$$

where $F(x)$, $G(x)$ are recursive functions and A,B,C are recursive terms, and the primitive recursive uniqueness rule

(U) $$\frac{F(Sx) = H(x,F(x))}{F(x) = H^xF(0)}$$

where the iterative function H^xt is defined by the primitive recursion $H^0t = t$, $H^{Sx}t = H(x,H^xt)$; in **U**, F may contain additional parameters.

In the same paper it is shown that the schema **U** may be replaced by

(E) $$\frac{F(0) = 0 \quad F(Sx) = F(x)}{F(x) = 0}$$

if we take as axioms

(A) $$a + (b \dot{-} a) = b + (a \dot{-} b)$$

and, in place of the introductory equations for the predecessor function,

(P) $$Sa \dot{-} Sb = a \dot{-} b$$

This system is referred to as $\mathbf{R}_1$.

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