Notre Dame Journal of Formal Logic
Volume VI, Number 2, April 1965

ON PROBABILITY LOGICS

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Our language contains the following symbols:

(1) the (individual) variables 'v_1', 'v_2', and so on;

(2) the sentential connectives '$\curlywedge$' ('not'), '$\rightarrow$' ('only if'), '$\wedge$' ('and'), '$\vee$' ('or'), and '$\leftrightarrow$' ('if and only if');

(3) the variable binders '$\mathbf{1}$' ('the'), '$\mathbf{P}$' ('the probability that any - is a ...'), '$\mathbf{Q}$' ('the probability that any - which is a ... is a ---'), '$\wedge$' ('for all'), and '$\vee$' ('for some');

(4) the individual constants '0', '1', 'c_3', 'c_4', and so on;

(5) the 1-place operation symbols '$\dot{-}$' ('minus'), '0_2^1', '0_3^1', and so on;

(6) the 2-place operation symbols '+' ('plus'), '.' ('times'), '-' ('minus'), '/' ('divided by'), '$\urcorner$' ('to the power'), 'Γ' ('the -th non-negative root of'), '0_7^2', '0_8^2', and so on;

(7) the 3-place operation symbols '0_1^3', '0_2^3', and so on; and so on for any greater number of places;

(8) the 1-place predicates '$\mathbf{R}$' ('is a real number'), '$\mathbf{N}$' ('is a positive integer'), 'P_3^1', 'P_4^1', and so on;

(9) the 2-place predicates '$\mathbf{I}$' ('is identical with'), 'α' ('is less than'), '$\mathbf{P}_3^2$' '$\mathbf{P}_4^2$', and so on; and

(10) the 3-place predicates '$\mathbf{P}_1^3$', '$\mathbf{P}_2^3$', and so on; and so on for any greater number of places.

We use the symbols '$<$', '$>$' and '$\{$', '$\}$' in the metalanguage to mark the boundaries of non-empty finite sequences and sets respectively. The letter 'm' will be used as a metalinguistic variable ranging over positive integers. Terms and formulas will be understood as follows:

(1) all variables and individual constants are terms;

(2) for any m-place operation symbol o and m-term sequence of terms t, $<ot>$ is a term;

(3) for any variable v and formulas f and g, $<$'$\mathbf{1}$' $vf>$, $<$'$\mathbf{P}$' $vf>$, and $<$'$\mathbf{Q}$' $vfg>$ are terms;

(4) for any m-place predicate p and m-term sequence of terms t, $<pt>$ is a formula;

(5) for any formulas f and g, $<$'$\curlywedge$' $f>$, $<f$ '$\rightarrow$' $g>$, $<f$ '$\wedge$' $g>$, $<f$ '$\vee$' $g>$, and $<f$ '$\leftrightarrow$' $g>$ are formulas; and

Received April 22, 1962