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AN AUGMENTED MODAL LOGIC

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The purpose of this paper is to examine the extent to which an augmented modal logic may be used as the formalized meta-theory $\mathfrak{M}$ of a formal system $\mathcal{S}$. The conditions which $\mathcal{S}$ must satisfy are indicated by Roman numerals. Assumptions and theorems of $\mathfrak{M}$ are indicated by Arabic numerals. Comments and examples are enclosed in square brackets.

I Let $\mathcal{S}$ consist of the following: (i) a set **V** of symbols, $v^1, v^2, \ldots v^j, \ldots$, (the vocabulary of $\mathcal{S}$); (ii) formation rules; (iii) axioms; (iv) transformation rules.

II Let **F** be the set of formulae $f^1, f^2, \ldots f^j, \ldots$, of $\mathcal{S}$, where each f^i consists of a finite string of symbols v^i.

III Let the following sub-sets of **F** be selected as follows:

(i) A set **W** of wff selected recursively from **F** by the formation rules.
(ii) A set **A** (the axioms of $\mathcal{S}$) selected recursively from **W** (commonly by giving a finite list).
(iii) A set **P** of provable formulae; a set **D** of disprovable formulae, a set **I** of irresoluble formulae; selected (but not necessarily recursively) from **W** by the transformation rules. Let **P** contain **A**.

IV Let the operations required to select the various sub-sets of **F** be entirely formal. [By this we mean that they depend only on the physical characteristics--shape, position, etc.,--of the v^i].

V Let the sets **P**, **D** and **I** satisfy the following conditions:

(i) If, for a given j, f^j occurs in **P**, then for some definite symbol, v^k say, $v^k f^j$ occurs in **D**; and conversely, if f^j occurs in **D** then $v^k f^j$ occurs in **P**. [Commonly v^k will be the symbol '$\sim$' (interpretable as 'not') and we say that f^j and $v^k f^j$ is each the negation of the other. We leave it open at this stage whether **P** (or **D**) can contain both f^j and $v^k f^j$, for a given j].
(ii) If neither f^j nor $v^k f^j$ occurs in **P** (in which case, by (i) above, neither occurs in **D**) then both occurs in **I**.

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