54
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THE POST-LINEAL THEOREMS FOR ARBITRARY RECURSIVELY ENUMERABLE DEGREES OF UNSOLVABILITY*

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Introduction

In 1949 Post and Lineal announced the existence of partial propositional calculi with unsolvable decision problems for theoremhood, completeness and independence of axioms. Unfortunately, they never published the proofs but only indicated in an abstract the idea of how a proof might be given. In 1963, Yntema and Harrop, independently, supplied proofs. As indicated in the abstract and carried out by Yntema, the main idea of the proof is that a semi-Thue system can be represented in a particular partial system in such a way that the word problem for the semi-Thue system is reducible to the decision problem for the partial system. Since the word problem for Thue systems is unsolvable as shown by an earlier theorem of Post (1947), the unsolvability of the corresponding decision problems for the partial systems then follows.

Post's earlier work (1944) on recursively enumerable sets of positive integers discussed recursive sets (those with solvable decision problem) and complete sets (those with decision problem of the highest degree of unsolvability) and tried to find sets whose decision problem was unsolvable but not of the highest degree. Although he did not succeed in this effort, since then and since the Post-Lineal theorems first appeared, Friedberg (1957) and Mučnik (1958) have found such sets. Consequently, old problems which have already been shown to be unsolvable are being reconsidered in the hope of proving that for each recursively enumerable degree of unsolvability there is such a problem.

One such problem is the word problem for Thue systems and Boone has proved that for every recursively enumerable degree of unsolvability there is a Thue system whose word problem is of that degree (1962).[1] The

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1. There are similar results by A. A. Fridman, G. S. Cetin and by C. R. L. Clapham the proofs of which are unknown to us.

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