

## MODAL SYSTEM S4.4

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It is known that Group II of Lewis-Langford, *cf.* [3], p. 493, i.e. the matrices  $\mathfrak{M}1$  and  $\mathfrak{M}2^1$

$\mathfrak{M}1$	*	$C$	$I$	$2$	$3$	$4$	$N$
		1	1	2	3	4	4
		2	1	1	3	3	3
		3	1	2	1	2	2
		4	1	1	1	1	1

$\mathfrak{M}2$	*	$p$	$M$	$L$
		1	1	1
		2	2	4
		3	1	3
		4	4	4

$\mathfrak{M}3$	*	$p$	$M$	$L$
		1	1	1
		2	1	4
		3	1	4
		4	4	4

which falsify the proper axiom of S5:

$$C11 \quad \mathfrak{C}MpLMp \quad (\text{i.e. } C11^* \quad \mathfrak{C}MLpLp)$$

are such that besides system S4, they verify several consequences of S5 which are unprovable in the former system, as, e.g., the formulas:

$$\begin{aligned} G1 & \quad \mathfrak{C}MLpLMp \\ D2 & \quad ALCLpqLCLqp \\ M1 & \quad \mathfrak{C}\mathfrak{C}\mathfrak{C}pLpLpCMLpLp \\ N1 & \quad \mathfrak{C}\mathfrak{C}\mathfrak{C}pLppCMLpp \end{aligned}$$

The theses  $G1$  and  $D2$  are the proper axioms of the well-known systems S4.2 and S4.3 respectively, *cf.* [2], [1], [6], and [11]. In [2], p. 263, Dummett and Lemmon have proved that  $M1$ , i.e. their formula (8), does not hold in S4.3. Prior, [6], p. 139, pointed out that Geach showed that in the field of S4.2 theses  $M1$  and  $N1$  are equivalent.

As one can easily notice  $\mathfrak{M}1$  and  $\mathfrak{M}2$  verify also the following two formulas

$$R1 \quad \mathfrak{C}pCMLpLp \quad (\text{i.e. } R1^* \quad \mathfrak{C}NpCMpLMp)$$

and

$$V1 \quad ALpALCpqLCPNq$$

It is clear that  $R1$  is a weaker form of  $C11^*$  (i.e. of  $C11$ ), but, as  $\mathfrak{M}1$  and  $\mathfrak{M}2$  show, in the field of S4 it does not imply S5. On the other hand