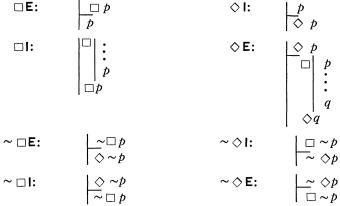
POSSIBILITY-ELIMINATION IN NATURAL DEDUCTION

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F. B. Fitch's extension of the subordinate-proof technique to modal logic¹ represents an interesting and valuable contribution to both study and exposition in the field. The modal introduction and elimination (intelim) rule-schemata he offers are these:



If the propositional base, to which it is understood that these rules are appended, is classical, then a system similar to Lewis' S4 is obtained by permitting only propositions of the form $\Box p$ (or $\sim \Diamond p$) to be reiterated into the strict subordinate proofs of $\Box I$ and $\Diamond E$. A weaker system similar to S2 is obtained by requiring such a reiterated proposition to drop its left-most modal operator.²

Two peculiarities, related in part to Fitch's restricted form of \sim I, emerge upon consideration of his modal rules. (1) Even on a classical base (which will be assumed throughout), the last four rules—those relating \square and \diamondsuit —cannot be derived from the first four—the fundamental intelim rules for \square and \diamondsuit ; and they are thus needed to complete the modal apparatus, (2) \square E and \diamondsuit I can be derived from each other, and \diamondsuit E from \square I (in the appropriate forms determined by the definition of \square p as \sim \diamondsuit \sim p and \diamondsuit p as \sim \square \sim p). But \square I in the form