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TWO REMARKS CONCERNING MENGER'S AND SCHULTZ' POSTULATES FOR THE SUBSTITUTIVE ALGEBRA OF THE 2-PLACE FUNCTORS IN THE 2-VALUED CALCULUS OF PROPOSITIONS.

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We denote the sixteen 2-place functors of the 2-valued calculus of propositions by the symbols introduced by Menger:[1]

$$1, A, B, C, D, E, I, J, 1' \ (=0), A', B', C', D', E', I', J'.$$

A is incompatibility, B is disjunction, C is implication, D is the converse implication, E is equivalence, I and J are the selectors, 1 is the constant functor of value 1. Primes indicate negation. The two hundred fifty-six 2-place transformations (i.e., ordered pairs of 2-place functors) constitute a semigroup which is isomorphic to the semigroup of all functions mapping $\{1,2,3,4\}$ into $\{1,2,3,4\}$.

Menger and Schultz show that this semigroup can be generated by the three transformations

$$h = (J', I'),\ e = (I', E),\ a = (A, I')$$

The first two transformations, by themselves, generate a subgroup isomorphic to the symmetric group on four elements.

Our first remark is that while h, e, and a are in terms of four functors (viz., I', J', E, A) the semigroup of the 2-place transformations in terms of only three 2-place functors, e.g. by either one of the following triples of transformations in terms of I', E, A.

$$e = (I',E),\ c = (E,I'),\ a = (A,I') \ \textit{or}\ e = (I',E),\ c = (E,I'),\ b = (A,E).$$

All that has to be shown in order to prove the first contention is that h can be expressed in terms of e and c. But this is the case since $h = e\ e\ c\ e$. The second contention can be established by the following expression of a in terms of e, c, and b:

$$a = e\ c\ b\ c\ c$$

It should be noted that

$$h\ h\ h = h,\ c\ c\ c\ c = c,\ e\ e\ e\ e\ e = e.$$

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