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ON NOT STRENGTHENING INTUITIONISTIC LOGIC

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We wish to reexamine —in the wake of R. E. Vesley's [7] —the question of converting so-called structural and intelim rules for PC_I, the intuitionistic sequenzen-kalkül of Gentzen, into rules for PC_C, the classical sequenzen-kalkül. We shall limit ourselves here to sequenzen or turnstile statements of the form $A_1, A_2, \ldots, A_n \vdash B$, where $A_1, A_2, \ldots, A_n$, $(n \geqslant 0)$, and B are wffs consisting of propositional variables, zero or more of the connectives '&', 'v', '∼', '⊃', and '≡', and zero or more parentheses.

One can pass from PC_I to PC_C by amending the intelim rules for '∼', a result of long standing, or by amending the intelim rules for either one of '⊃' and '≡', a more recent find.[1] In a talk at Yale University in 1961, however, Leblanc conjectured that amending the intelim rules for either one of '&' and 'v' will not do the trick. The point, mentioned in Leblanc [4], appears as follows in Leblanc and Belnap [5]:

> We also conjecture, by the way, that any structural rule which holds in PC_C also holds in PC_I, that any elimination or introduction rule for '&' and 'v' which holds in PC_C also holds in PC_I, and hence that the only way of turning standard Gentzen rules of inference for PC_I into rules for PC_C is to strengthen the elimination or introduction rules for '∼' or those for '⊃', or those for '≡'.

Leblanc's conjecture, to which we devote the rest of this paper, has had a rather checkered career: proved true at one time or another by three different writers in two different ways, it has also been proved false once.[2] To resolve this seeming contradiction and sort out what has been proved true and what false, we shall have another look at some of the key terms in the above quotation. It will turn out that the readings of Leblanc's conjecture in [1], [2], and [7] are not quite apposite, and that of two fresh ones which we consider here one is unrestrictedly true, while the other holds under a slight restriction.

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