224
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A STIPULATION OF A MODAL PROPOSITIONAL CALCULUS IN TERMS OF MODALIZED TRUTH-VALUES

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The logical truths of a non-modal propositional calculus may be identified as the theorems following from certain axioms by certain rules. Alternatively, they may be identified by means of two-valued truth-tables as those propositions whose truth-tables have only **T's** in the main column. Stipulations of the logical truths of modal propositional calculi have been given in terms of axioms and rules, but not, as far as I know, in terms of what I will call the modalized truth-values (**MTVs**), viz. logical truth, contingent truth, contingent falsity, and logical falsity. I attempt the latter type of stipulation in the present note, using as a guide throughout the two-valued truth-value stipulation.[1]

Let the object-language consist of a non-modal propositional calculus with '$\vee$' and '$\sim$' as primitive connectives, with propositional constants but no variables, to which is added the functor '4' with '$4(p)$' being well-formed if p is. The intended interpretation of '$4(p)$' is 'It is logically false that p' or '*It is logically impossible that* p'.

The definition of 'logical truth' will be given in terms of an alteration or rewriting of the usual tables of a four-valued logic. The four values will be represented by '1', '2', '3', and '4', corresponding respectively to logical truth, contingent truth, contingent falsity, and logical falsity. The basic matrices used are:

p	$\sim p$	$4p$
1	4	4
2	3	4
3	2	4
4	1	1

$\vee$	1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	2	3	3
4	1	2	3	4

I submit that the content of these matrices is intuitively reasonable, except for the places (2,3) and (3,2) in the disjunction matrix, to which I will return below. By saying they are intuitively reasonable, I mean that, for example, to assign the value 4, logical falsity, to the denial of a proposition whose **MTV** is 1, logical truth, accords with our intuitive handling of these concepts.

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