Notre Dame Journal of Formal Logic
Volume IV, Number 2, April 1963

A NOTE ON MODAL SYSTEMS

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In [2], § 4.2, p. 58, I have proved that in the customary axiomatization of Lewis' system S5 one axiom, viz. A7 (or B7) is superfluous, i.e., in the other words, that $\{S1^{\circ}; C11\} \rightleftarrows \{S5\}$. In [4] Thomas pointed out that the deductions due to which I obtained this result clearly show that the following metarule

$$\mathfrak{R} \quad \textit{If} \vdash \mathbb{S}M\alpha L\beta, \textit{ then} \vdash \mathbb{S}\alpha\beta$$

is derivable in $S1^{\circ}$. Since, as far as I know, this metarule $\mathfrak{R}$ and the various modal theses related to it were unobserved previously, it seems that the following brief remarks about them can be useful.

(i) Consider the following modal theses

R1 $CCMpLq\mathbb{S}pq$

R2 $CCMpLqCpq$

R3 $C\mathbb{S}MpLqCpq$

R4 $C\mathbb{S}MpLq\mathbb{S}pq$

R5 $\mathbb{S}CMpLqCpq$

R6 $\mathbb{S}\mathbb{S}MpLqCpq$

R7 $\mathbb{S}CMpLq\mathbb{S}pq$

R8 $\mathbb{S}\mathbb{S}MpLq\mathbb{S}pq$

each of which is in some way akin to metarule $\mathfrak{R}$, and, additionally, the theses

A7° $CpMp$

*A7** $CpMMp$

*A7*** $CMpMMp$

A7+ $\mathbb{S}pMMp$

A7++ $\mathbb{S}MpMMp$

which we shall need in our discussion.

Received February 27, 1963