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A SYSTEM OF QUANTIFICATIONAL DEDUCTION[1]

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I. *Introduction.* This paper will describe a simple system of quantificational deduction and give proofs of its soundness and completeness. The latter proof will be put in terms of a device to be introduced in a later section, the quantifier game. This expository gambit, which leads to a rather simple proof, permits an independent treatment of key semantic principles prior to the details of their application. Elsewhere, the device may be put to work both as a pedagogical ploy and in theoretical arguments, which lends it an interest that extends beyond the scope of its uses here. The present completeness proof carries over to most systems of natural deduction, since the deductions permitted by the system whose completeness is proved are easily seen to have counterparts in these other systems. However, this system is a clumsy one to use, and so for practical purposes its equivalence to a more workable system will be sketched in a final section.

II. *The system of deduction.* This system is designed to prove the inconsistency of single quantificational formulas in prenex normal form. It also provides for a *reductio ad absurdum* proof that an argument is valid, since it may be used to prove that a prenex normal form version of a conjunction of the premises and the negation of the conclusion is inconsistent.

The system has just two rules of derivation, called **UI** and **EI**. Let *Fm* be any formula in which *m* occurs free and let *Fn* be like *Fm* except that *Fn* has free *n* everywhere that *Fm* has free *m*. Then **UI** and **EI** are the rules whereby we respectively pass from a formula of form $(m)Fm$ or $(\exists m)Fm$ to the corresponding formula of form *Fn*. Starting with a single formula in prenex normal form, these rules enable us to write down a sequence of lines each

1. This system, which relates closely to the "more economical one" of Quine [3], p. 254, and, more remotely, to Method A of Quine [4], derives from Herbrand's Theorem, as may be seen from the version of this given in Hilbert and Bernays [2], pp. 157-163. For more light on these historical roots, also see Dreben [1]. I am grateful to J. S. Ullian and P. J. S. Benacerraf, who read an earlier draft of this paper and made helpful suggestions.

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