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PROOF ROUTINES FOR THE PROPOSITIONAL CALCULUS

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I prove in the pages that follow a conjecture of mine, to wit:

Any metastatement of the form

$$A_1, A_2, \ldots, A_n \vdash B,$$

where $A_1, A_2, \ldots, A_n (n \geq 0)$, *and* B *are wffs of* PC *and* '$\vdash$' *is the customary yields sign, is provable, when valid, by means of the three structural rules in Table I and the intelim rules in Table I for such of the connectives* '$\sim$', '$\supset$', '&', 'v', *and* '$\equiv$' *as occur in* $A_1, A_2, \ldots, A_n \vdash B$,

and sketch a routine for proving $A_1, A_2, \ldots, A_n \vdash B$, when valid, for each one of the 32 cases covered by the conjecture.[1] I also discuss a related conjecture of mine concerning the intuitionist fragment of PC.

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I

Let all five of '$\sim$', '$\supset$', '&', 'v', and '$\equiv$' be elected to serve as the primitive connectives of PC; let 'A', 'B', 'C', and 'D' be elected to range over the well-formed formulas (wffs) of PC; let a metastatement of the form $A_1, A_2, \ldots, A_n \vdash B$, called for short a T-statement, be rated valid if, in case $n = 0$, B is satisfied by any assignment of truth-values to the propositional variables occurring in B, or, in case $n > 0$, B is satisfied by any assignment of truth-values to the propositional variables occurring in A_1, $A_2, \ldots, A_n$, and B which simultaneously satisfies $A_1, A_2, \ldots,$ and A_n; let a T-statement be rated provable if it is the last entry in a finite column of T-statements each one of which is of the form **R** in Table I or follows from one or more previous T-statements in the column by application of one of the remaining rules in Table I; and, finally, let a T-statement be rated provable by means of the structural rules in Table I (to be collectively referred to as **S**) and zero or more of the intelim rules in Table I if it is the

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