Notre Dame Journal of Formal Logic
Volume IV, Number 1, January 1963

A NOTE ON THE GENERALIZED CONTINUUM HYPOTHESIS. II

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§2*

In [6] I have proved that $\mathfrak{A}$, i.e. the generalized continuum hypothesis, is equivalent to the following formula

A *For any cardinal numbers* $\mathfrak{m}$ *and* $\mathfrak{n}$ *which are not finite, if* $\mathfrak{n} < 2^{\mathfrak{m}}$, *then* $\mathfrak{n} \leqslant \mathfrak{m}$

The following convenient abbreviation defined inductively:

For any natural number n, $0 \leqslant n < \infty$, *and any cardinal number* $\mathfrak{m}$

$$\textit{symbol } \overset{\langle n\rangle}{2}\,\mathfrak{m} \textit{ means } \begin{cases} \textit{if } n = 0, \textit{ then } \overset{\langle n\rangle}{2}\,\mathfrak{m} = \mathfrak{m} \\ \textit{if } n > 0, \textit{ then } \overset{\langle n+1\rangle}{2}\,\mathfrak{m} = 2^{\overset{\langle n\rangle}{2}\mathfrak{m}} \end{cases}$$

allows us to express the formulas $\mathfrak{A}$ and **A**, as follows

$\mathfrak{A}(= \mathfrak{A}_0)$ *If* $\mathfrak{m}$ *is a cardinal number which is not finite, then there exists no* $\mathfrak{n}$ *such that* $\overset{\langle 0\rangle}{2}\,\mathfrak{m} < \mathfrak{n} < \overset{\langle 1\rangle}{2}\,\mathfrak{m}$

$\mathbf{A}(= \mathbf{A}_0)$ *For any cardinal numbers* $\mathfrak{m}$ *and* $\mathfrak{n}$ *which are not finite, if* $\mathfrak{n} < \overset{\langle 1\rangle}{2}\,\mathfrak{m}$, *then* $\mathfrak{n} \leqslant \mathfrak{m}$,

and their particular instances which we obtain by putting $2^{\mathfrak{m}}$, $2^{2^{\mathfrak{m}}}$, $2^{2^{2^{\mathfrak{m}}}}$ *etc.* for $\mathfrak{m}$ in $\mathfrak{A}$ or in **A**, as

*The first part of this paper appeared in *Notre Dame Journal of Formal Logic*, v. III (1962), pp. 274-278. It will be referred to throughout this second part, as [7]. See additional Bibliography given at the end of this part. An acquaintance with [7] is presupposed.

Received August 16, 1962