A DIAGRAM OF THE FUNCTORS OF THE TWO-VALUED PROPOSITIONAL CALCULUS

THOMAS W. SCHARLE*

By means of arranging the functors of the two-valued propositional calculus in a certain array (to be described below), we find that several properties of the functors are related. Such properties are connected to the possibilities of defining some functors by others, and thus in the diagram we have displayed definitional connections between certain sets of functors. In this paper we first present the method of diagramming, and certain helpful connections within the diagram, then several theorems on definitions within the propositional calculus. We are then able to show that there are three exhaustive classes for single functors in terms of definitions, of such a nature that we are able to give axioms for a large number of functors. The paper is concluded with some further consideration on definability in special cases.

Let us arrange the unary functors of the two-valued propositional calculus[1] in the array $\mathfrak{A}_1$.

$$
\begin{array}{ccc}
 & I & \\
 \mathfrak{A}_1 \qquad\qquad\qquad O & & U \\
 & N &
\end{array}
$$

We may then extend this array to include also the binary functors by placing a given binary functor, X, having the properties, that, for certain unary functors Y and Z,

$$Xpp = Yp$$
$$XpNp = Zp$$

in an array similar to $\mathfrak{A}_1$, and half its size, centered at the functor Y, with X at the position corresponding to that occupied by Z in the array $\mathfrak{A}_1$. This

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