

ON THE GENERALIZED BROUWERIAN AXIOMS

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After Oskar Becker¹ a modal thesis of the following form:²

$$B_n \quad \mathcal{C}pL^nMp$$

for any $n > 1$, is called a generalized Brouwerian axiom. Since in Lewis' system S4 the following thesis

$$M1 \quad \mathcal{C}LpLLp$$

holds, it is obvious that in S4 (and hence a fortiori in S5) every formula B_n , for any $n > 1$, is inferentially equivalent to the proper Brouwerian axiom, i.e. Lewis' thesis

$$C12 \quad \mathcal{C}pLMp$$

On the other hand, it seems that in the field of some of Lewis' systems which are weaker than S4, a generalized Brouwerian axiom B_n , for any $n > 1$, is a stronger thesis than C12. For while, as far as I know, only the following definitive results concerning the addition of C12 to the systems weaker than S4 are obtained:

- a) In [5], pp. 151-152, Parry has proved that the addition of C12 to S3 gives system S5 of Lewis.

and

- b) In [8], pp. 56-58, I have shown recently that the same holds, if we add C12 as a new axiom either to S3^o or to S3*.

and while the effect of the addition of C12 either to S1^o or to S1 is not yet fully investigated,³ in [2], pp. 78-81, it is proved by Churchman that the addition of B_n , for any $n > 1$, to S2 gives system S5.

In this note I shall investigate some properties of a generalized Brouwerian axiom, i.e. of formula B_n , for any $n > 1$. Namely:

- 1) In §1 a certain subsystem of S1 is defined. This system, called S1* is such that it is weaker than S1, it contains S1^o and it is stronger than the latter system.