CORRELATIVE REMARKS CONCERNING ELEMENTARY NUMBER THEORY, GROUPS AND MUTANT SETS

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0. Introduction. The philosophical works coming from the Pythagoreans and from the logical investigations of Aristotle, to cite some ancient philosophers, emphasize hosts of dichotomies used by man's reasoning faculties, e.g., odd and even, finite and infinite and male and female. Kant, Hegel and C. S. Peirce, among others, sought to develop parts of their philosophies around triads compounded of dichotomies carried forward to a synthesis of sorts.

With a similar spirit in mind this brief note gives a general theorem dealing with closed and anti-closed (see references [1], [2]) subsets of general algebraic systems. In particular the theorem shows a logical relation between naive (i.e., non-axiomatic) algebraic number theory and such mutant (i.e., anti-closed) sets of general algebraic systems.

1. Proposition. A (λ, T)-*mutant* of a general algebraic system $(A, *)$ is a subset M of A which satisfies the condition that $\{a_1 * a_2 * \ldots * a_\lambda : a_1 \in M, \ a_2 \in M, \ldots, a_\lambda \in M\} \subseteq T \cap \overline{M}$, where λ is an integer greater than or equal to 2, T together with $*$ forms a general algebraic subsystem of $(A, *)$ and $\overline{M}$ is the set-theoretic complement of M. For a fixed λ and T, a (λ, T)-*mutant* of $(A, *)$ is said to be a *maximal* (λ, T)-*mutant* of $(A, *)$ provided there is no (λ, T)-*mutant* of $(A, *)$ properly containing it.

Theorem: Put $\mathbf{G}(\lambda) = \{2^\lambda m : m = 0, \pm 1, \pm 2, \ldots\}$, where λ is a non-negative integer. Consider the additive groups $(\mathbf{G}(\lambda), +)$ of all 2^λ multiples of the set of all integers. Then for every non-negative integer λ there exist a *maximal* $(2, \mathbf{G}(\lambda + 1))$-*mutant* M_λ of $(\mathbf{G}(\lambda), +)$ and a *maximal* $(2, \mathbf{G}(\lambda + 2))$-*mutant* M'_λ of $(\mathbf{G}(\lambda), +)$.

Proof: Every general algebraic system has a mutant and a maximal mutant [2]. It is trivial to establish the existence of M_λ. Thus, recall that (i) the property of being a maximal mutant is preserved under isomorphism [3], that (ii) the set of all odd integers is a *maximal* $(2, \mathbf{G}(1))$-*mutant* of $(\mathbf{G}(0), +)$ and that (iii) the $(\mathbf{G}(\lambda), +)$ are isomorphic. It is somewhat less trivial to establish the existence of M'_λ. Once again make use of (i) and

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