ON THE FORMALIZATION OF TWO MODAL THESES

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I. In medieval times, when a flowering of modal logic was in progress among scholastic logicians, two modal theses were formulated, and accorded widespread acceptance:

> (T1) The "mere possibility" of a proposition cannot entail its factuality (*a posse ad esse non valet consequentia*).

> (T2) No "mere fact" or "merely contingent proposition" entails a necessary proposition (*a esse ad necesse non valet consequentia*).

It is the objective of the present paper to examine the issues raised in the questions: How are these theses to be articulated within the framework of modern formalizations of modal logic? What symbolic rendition is appropriate for them? What special assumptions, if any, are requisite if the appropriate formalized versions of these theses are to enjoy the status of acceptable modal principles?

II. For the purposes of the present discussion, we assume a symbolic system of modal logic based on the operators "$\Diamond$", "$\Box$", and "$\rightarrow$" (representing possibility, necessity, and strict implication, respectively). This modal system is assumed to be "normal" in the sense that at least the following definitions and laws obtain, in addition to *modus ponens* and a substitution:

(D1) $\Box p = Df \sim \Diamond \sim p$

(R1) $\Box p \rightarrow p$

(R2) $(p \ \& \ q) \rightarrow p$

(R3) $\Diamond (p \ \& \ q) \rightarrow \Diamond p$

(R4) $(p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$

(R5) $(p \rightarrow q) \rightarrow \sim \Diamond (p \ \& \sim q)$

Derivatively (in view of *D1* and *R1*) we have:

(R6) $p \rightarrow \Diamond p$