ON ŁUKASIEWICZ'S Ł-MODAL SYSTEM

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I want to provide a proof of Łukasiewicz's assertion that his Ł-modal system[1] is characterised by a four-valued matrix. The need for such a proof was pointed out to me by Dr. R. Harrop.

The theorems of the Ł-modal system are the formulae that can be derived from the axioms

1. $C\delta pC\delta Np\delta q$,

2. $Cp\Delta p$,

by use of the rules of substitution (for propositional and functorial variables) and detachment.

The matrix in question (call it **M**) is the cross-product of a pair of two-valued matrices:

$\mathbf{M}_1$:

	C	t	f	N	Δ
*	t	t	f	f	t
	f	t	t	t	f

$\mathbf{M}_2$:

	C	t	f	N	Δ
*	t	t	f	f	t
	f	t	t	t	t

If for convenience we number the elements of the product, writing $<t, t> = 1$, $<t, f> = 2$, $<f, t> = 3$, $<f, f> = 4$, we can then describe **M** as follows:

	C	1	2	3	4	N	Δ
*	1	1	2	3	4	4	1
	2	1	1	3	3	3	1
	3	1	2	1	2	2	3
	4	1	1	1	1	1	3

It is easy to show that every theorem is verified by (never takes an undesignated value in) **M**. We want to prove the converse, that every formula verified by **M** is a theorem. We first observe that the truth-table of Δ in the matrix $\mathbf{M}_1$ is such that a formula $\Delta\alpha$ always takes the same truth-value as α itself. If then β is any formula, and β_1 is got from it by replacing

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