ON THE SINGLE AXIOMS OF PROTOTHETIC

BOLESŁAW SOBOCIŃSKI

§6*. When in 1938 I proved that condition $\mathfrak{b}$ of metarule **L** follows from the remaining conditions $\mathfrak{a}$ and $\mathfrak{c}$[74], I also remarked that probably condition $\mathfrak{a}$ of **L** could be weakened too. Viz., I thought that system $\mathfrak{S}$ could be substituted by a suitable fragment of it, and that this would certainly be done, if we had at our disposal *SIII*, which as we remember enables us to make extensional deductions for expressions belonging to the semantical category of propositions. This suggestion of mine and the result concerning the redundancy of condition $\mathfrak{b}$ enabled Leśniewski to construct his axiom A_m:[75]

$$A_m \qquad [p\ q] \mathbin{::} p \equiv q \,.\! \equiv \mathbin{:\cdot:} [f] \mathbin{:\cdot:} f(q\,f(q\,[u]\,.\,u))\,.\! \equiv \mathbin{::} [r] \mathbin{::} f(p\,r)\,.\! \equiv \mathbin{.:.} r \equiv : q \equiv .\ r \equiv p$$

In fact, starting from A_m alone and using deductions in some respects similar but less complicated than those presented in §4 of this chapter one can easily prove:

$$P1 \qquad [p\ q] \mathbin{.:.} p \equiv : q \equiv .\ p \equiv q$$

A13, and metarules *SII*, *SIII* and *SV* which amounts to condition $\mathfrak{c}$. Now, given the rule of $\mathfrak{S}_5$ *P1*, *A13*, *SII*, *SIII* and *SV* suffice for the purpose of deriving *A78*, i.e. condition $\mathfrak{a}$. Moreover, this can be done in a very elementary way. These deductions of Leśniewski confirmed the correctness of my remark. For, obviously, in the set of assumptions $\{A13;\ P1;\ SII;\ SIII;\ SV\}$ we can replace *SII* by *A76*, but on the basis of the rule of procedure of $\mathfrak{S}$ the theses *A13*, *A76* and *P1* do not constitute a complete system of $\mathfrak{S}$. We can prove it using the following matrix:

*The first two parts of this paper appeared in *Notre Dame Journal of Formal Logic*, vol. I (1960), pp. 52-73 and vol. II (1961), pp. 111-126. They will be referred throughout the remaining parts, as [35] and [36]. See additional Bibliography given at the end of this part.

Received August 3, 1960