STUDIES IN THE AXIOMATIC FOUNDATIONS OF BOOLEAN ALGEBRA

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Section IV

In Section II we made use of the rule for writing propositional definitions in order to define singular inclusion in terms of weak inclusion within the framework of $\mathfrak{S}$ (or $\mathfrak{A}^*$).[11] Our definition had the form of the following expression:

$D6.$ $[a\ b] :\cdot: a \in b . \equiv : : [\exists c] . \sim (a \subset c) . a \subset b : : [c\ d] .\cdot. c \subset a . \supset : a \subset c . \vee . c \subset d$

It is obvious that $D6$ can also serve as a definition of singular inclusion within the framework of $\mathfrak{A}$.

One might expect, on purely intuitive grounds, that the familiar proposition

$D_0D1.$ $[a\ b] .\cdot. a \subset b . \equiv : [c] : c \in a . \supset . c \in b$

could in turn be used as a definition of weak inclusion in terms of singular inclusion, and that the functor of singular inclusion could be employed as a primitive constant term in a system of Boolean Algebra with definitions. Interestingly enough D_0D1 does not seem to be derivable within $\mathfrak{A}$ unless we strengthen

$A1.$ $[a\ b] .\cdot. a \subset b . \equiv : [c\ d\ e] : \sim (c \subset d) . c \subset e . c \subset a . \supset . [\exists f\ g] . \sim (f \subset g) . f \subset e . f \subset b$

by subjoining to it

$A1.1$ $[a\ b] :\cdot: \sim (a \subset b) . \supset : : [\exists c\ d] : : \sim (c \subset d) . c \subset a : : [e\ f] .\cdot. e \subset c . \supset : c \subset e . \vee . e \subset f$

Within the framework of $\mathfrak{A}$ proposition $A1.1$ appears to be independent of proposition $A.1$. This statement, however, will have to be regarded as a conjecture until an interpretation is found which satisfies $A1$ and the rules of $\mathfrak{A}$, including the rule for writing nominal definitions, but fails to satisfy $A1.1$.[12]

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