ON CHARACTERIZATIONS OF THE FIRST-ORDER FUNCTIONAL CALCULUS

JULIUSZ REICHBACH

In papers [5] and [7][1] I have presented some characterizations of theses of the first-order functional calculus; in this paper I give a generalization of two characterizations of one.

We consider the first-order functional calculus with the symbolism described in [4][2] and besides signs accepted in the logic literature we use the following ones:

(0,1) $E, F, G, E_1, F_1, G_1 \ldots$ – variables representing expressions,

(0,2) $Sw\{E\}$ – the set of all symbols occurring in the expression E,

(0,3) Skt – the set of all formulas[3] of the form $\Sigma a_1 \ldots \Sigma a_i \Pi a_{i+1} \ldots \Pi a_k F$,[4] where F is a quantifierless expression containing no free variables and Πa_j is the sign of the universal quantifier binding the apparent variable a_j, and $\Sigma a_j G = (\Pi a_j G')'$, for $j = 1, \ldots, k$.

(0,4) $C(E)$ – the set of all significant parts of the formula E: $F \epsilon C(E) \cdot \equiv \cdot F = E$ or there exist such G, H that: $(F = G) \wedge (E = G') \vee [(F = G) \vee (F = H)] \wedge (E = G + H) \vee (\exists i)\{F = G(x_i/a)\} \wedge (E = \Pi a G)$.[5]

(0,5) $w(E)$ – the number of different free variables occurring in the expression E,

(0,6) $p(E)$ – the number of different apparent variables occurring in the expression E,

(0,7) $i_1, \ldots, i_{w(E)}$, or $j_1, \ldots, j_{w(E)}$ or $l_1, \ldots, l_{w(E)}$ – different indices of these and only these free variables which occur in the expression E,

(0,8) $i(E) = \max\{i_1, \ldots, i_{w(E)}\}$,

(0,9) $m(E) = w(E) + p(E)$,

(0,10) $n(E) = \max\{m(E), i(E)\}$,

(0,11) $E(x/y)$ – the expression resulting from E by the substitution of x for each occurrence of y in E; if y is an apparent variable, then y does not belong in E to the scope of the quantifier Πy; if x is an apparent variable, then y does not belong to the scope of the quantifier Πx,

(0,12) $\Sigma(F) = 0$, if F is a quantifierless formula; $\Sigma(F + G) = \max\{\Sigma(F), \Sigma(G)\}$; $\Sigma(\Pi a F) = \Sigma\{F(x/a)\}$, where $x \overline{\epsilon} Sw\{F\}$; $\Sigma(\Sigma a F) = w(F) + 1$, if $\Sigma\{F(x/a)\} = 0$;[6] $\Sigma(\Sigma a F) = \Sigma\{F(x/a)\}$, if $x \overline{\epsilon} Sw(F)$ and $\Sigma\{F(x/a)\} \neq 0$;

Received April 7, 1960