# INDEPENDENCE OF TARSKI'S LAW IN HENKIN'S PROPOSITIONAL FRAGMENTS

IVO THOMAS

The system $\{A1\text{-}3,\ (\varphi)^*\}$ is the system of Henkin's [1], proved by him complete for tautologies in implication and whatever truth-function $\varphi(x_1, \ldots, x_m)$ may be. If $m = 0$, $\varphi$ is just T or F. The basis of the system is *modus ponens*, the axiom schemata

$$\begin{aligned} &A1. \quad A \supset . B \supset A \\ &A2. \quad A \supset B \supset . A \supset (B \supset C) \supset . A \supset C \\ &A3. \quad A \supset C \supset . A \supset B \supset C \supset C \end{aligned}$$

and a set, $(\varphi)^*$, of $2^m$ axiom schemata

$$x_1^* \supset . x_2^* \supset . \ldots \supset . x_m^* \supset \varphi'^*$$

in which $x_i^*$ is $x_i$ or $x_i \supset y$ (with $y$ a new variable) in the $j$-th schema according as $x_i$ is T or F in the $j$-th valuation (according to some ordering) of $\varphi(x_1, \ldots, x_m)$, and $\varphi^*$ is $\varphi(x_1, \ldots, x_m) \supset y \supset y$ or $\varphi(x_1, \ldots, x_m) \supset y$ according as $\varphi(x_1, \ldots, x_m)$ is T or F. (Henkin used $x_i \supset y \supset y$ in place of our antecedents $x_i$, but since $A \supset . B \supset C$ and $A \supset C \supset C \supset . B \supset C$ are equivalent forms in any system containing $A1\text{-}2$, we use the shorter expression.) $\varphi$ is a function symbol, but we shall usually refrain from indicating its argument places, and this should not cause confusion.

L'Abbé in [2] showed that only the independence of $A3$ is ever in doubt. We here show the general (necessary and sufficient) conditions for $A3$ to be independent[1]), the method of determining this being simple inspection of a truth-table for $\varphi$. The term 'Tarskian' in the ensuing theorem is chosen because $A3$ is the often so-called Law of Tarski with commuted antecedents.

*Def.* $\mathfrak{T}$ For all $\varphi$, $\varphi$ is Tarskian iff there are valuations of $\varphi$, say $\alpha$ and $\beta$, such that $\varphi$ is F in $\alpha$, T in $\beta$, and all arguments of $\varphi$ that are T in $\beta$ are T in $\alpha$.

THEOREM. $A3$ is independent in the system $\{A1\text{-}3,\ (\varphi)^*\}$ iff $\varphi$ is not Tarskian.

[1] We are indebted to Professor Henkin for suggesting this problem.

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