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DECISION PROBLEM IN THE CLASSICAL LOGIC

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The important problem of decision in mathematical logic has been studied by many authors; it was resolved for propositional calculus. For functional calculus, Church demonstrated that there was no model by which we can determine whether a well-formed formula of the predicate calculus is true or false.

In classical logic a formula "α" is a tautology if for the propositional variables:

$$p_1, p_2, \ldots, p_n$$

in "α" we can make correspond the truth values:

$$a_1, a_2, \ldots, a_n$$

(where each of "a_i" are constant, $\mathbf{v} = true$, and $\mathbf{f} = false$) and the substitution of the variable p_i by a_i conduct to "α" true). In our paper we shall say that "α" is a tautology if its logical value is $\mathbf{v}$, where *logical value* of a formula means the result which we get making the substitution of the propositional variables by $\mathbf{v}$ or $\mathbf{f}$ in all possible ways and making all the operations connected.

The purpose of this article is to present a new method for the resolution of the decision problem, a method which is an immediate result of our studies on normal forms in propositional calculus. The work is treated in this way: I. For forms made with equivalence. II. For forms made with equivalence, negation, reciprocity. III. For forms made with equivalence, reciprocity and alternation. IV. For a general form of classical logic.

For all these we use the notation of *J. Łukasiewicz*. The idea of form is defined in this way:

1. Each propositional variable is a form;
2. If "α" is a form and "F" is a unary functor, then "$F\alpha$" is a form;
3. If "α" and "β" are forms and "F" is a binary functor, then "$F\alpha\beta$" is a form. The set of the forms made by the means of the functors $F_1, F_2, \ldots, F_n$ is to be written: $\mathbf{S}(F_1, F_2, \ldots, F_n)$. For simplicity, we denote by S the set of all forms from classical logic. Two forms "α" and "β" are equipollent

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